PROBING ACOUSTIC NONLINEARITY BY MIXING SURFACE ACOUSTIC WAVES

David H. Hurley
Ken L. Telschow

Idaho National Engineering and Environmental Laboratory
P.O. Box 1625
Idaho Falls, ID 83415-2209

ABSTRACT

Measurement methods aimed at determining material properties through nonlinear wave propagation are sensitive to artifacts caused by background nonlinearities inherent in the ultrasonic generation and detection methods. The focus of this paper is to describe our investigation of nonlinear mixing of surface acoustic waves (SAWs) as a means to decrease sensitivity to background nonlinearity and increase spatial sensitivity to acoustic nonlinearity induced by material microstructure.

INTRODUCTION

Ultrasonic techniques that investigate the nonlinear response of a material offer the potential for measuring certain material properties more effectively than techniques that assume linear behavior [1]. Consider an initially sinusoidal elastic wave propagating through a nonlinear medium. For a bulk wave analysis, an expression involving the amplitude ratio of the first overtone to the square of the fundamental, β, serves as a quantitative measure of the strength of harmonic generation. Microstructural features such as voids, dislocations and dislocations dipoles deform the crystal lattice causing changes in the magnitude of β [2-4]. Many researchers have related changes in β to changes in material microstructure. For instance, Cantrell and Yost [5] successfully predicted variations in β caused by precipitate coherency strains in the artificial aging of aluminum alloy 2024.

Most investigations involving analysis of nonlinear elastic coupling suffer from sensitivity to background nonlinearities such as those related to the electronic and electromechanical equipment [6]. Furthermore, methods that involve bulk wave analysis measure the average contribution to the nonlinear portion of the signal over a distance equal to the probe beam propagation distance [7]. In order to decrease sensitivity to background nonlinearities and increase spatial sensitivity we have investigated nonlinear mixing of surface acoustic waves of different frequencies, ω_1 and ω_2. If the difference frequency is chosen such that it is not equal to ω_1 or ω_2, a convenient parameter analogous to β can be defined. In addition, the wave guiding nature of SAWs allows the generation and detection process to be performed on the same side of the sample. This in turn affords enhanced spatial sensitivity since the acoustic propagation distance is not dictated by the sample's dimensions.

DEFINING A NONLINEARITY PARAMETER FOR SURFACE ACOUSTIC WAVES

An expression analogous to β for surface acoustic waves can be derived by considering the spatial evolution of the fundamental and higher harmonics. For the analysis presented in this paper, we used the theoretical approach developed by Zabolotskaya [8] for plane surface waves in an isotropic material. The initial particle velocity profile is represented by

$$V_0\big|_{x=0} = A_0 e^{i(\omega_0 t)} \ , \tag{1}$$

where ω_0 is the fundamental frequency. As the initial disturbance propagates, the wave from distorts and is represented by

$$\begin{aligned} V_x &= \sum_n V_n u_{xn}(z) e^{in(k_0 x - \omega_0 t)} \\ V_z &= \sum_n V_n u_{zn}(z) e^{in(k_0 x - \omega_0 t)} . \end{aligned} \tag{2}$$

The spatial evolution of the amplitude coefficients, V_n, is governed by

$$\frac{dV_n}{dx} = \frac{n^2}{8|S_{11}|} \sum_{m=1}^{n-1} S_{m,n-m} V_m V_{n-m} - 2 \sum_{m=n+1}^{N} S_{n,m-n} V_m V_{m-n} - \alpha_n V_n \ , \tag{3}$$

where the propagation distance, x, has been normalized by x_0:

$$x_0 = \frac{\rho \ c^4}{4 \ |S_{11}| \omega \ v_0} \ , \tag{4}$$

which can be thought of as the distance to shock. The nonlinear coupling matrix, $S_{m,n}$ is a function of both the second and third order elastic constants. The first summation in Eq. (3) corresponds to sum frequency generation, the second term corresponds to difference frequency generation and the last term on the right hand side accounts for frequency dependent attenuation.

In order to incorporate the effects of wave mixing into the present theory, the initial velocity profile must be expanded in a Fourier series about the difference frequency, $\Delta\omega=\omega_2-\omega_1$. In Figure 1, the numerical solution to equation (3) is graphically presented for both a single frequency source and a dual frequency source. First, note that the harmonic evolution is scaled by x_0 (fully developed nonlinear behavior arises more quickly for higher frequencies). For the single frequency source, the decrease in amplitude of the fundamental is caused both by attenuation and coupling to higher harmonics. For the dual frequency source, the two frequencies chosen where ω_1 and $2\omega_1$.

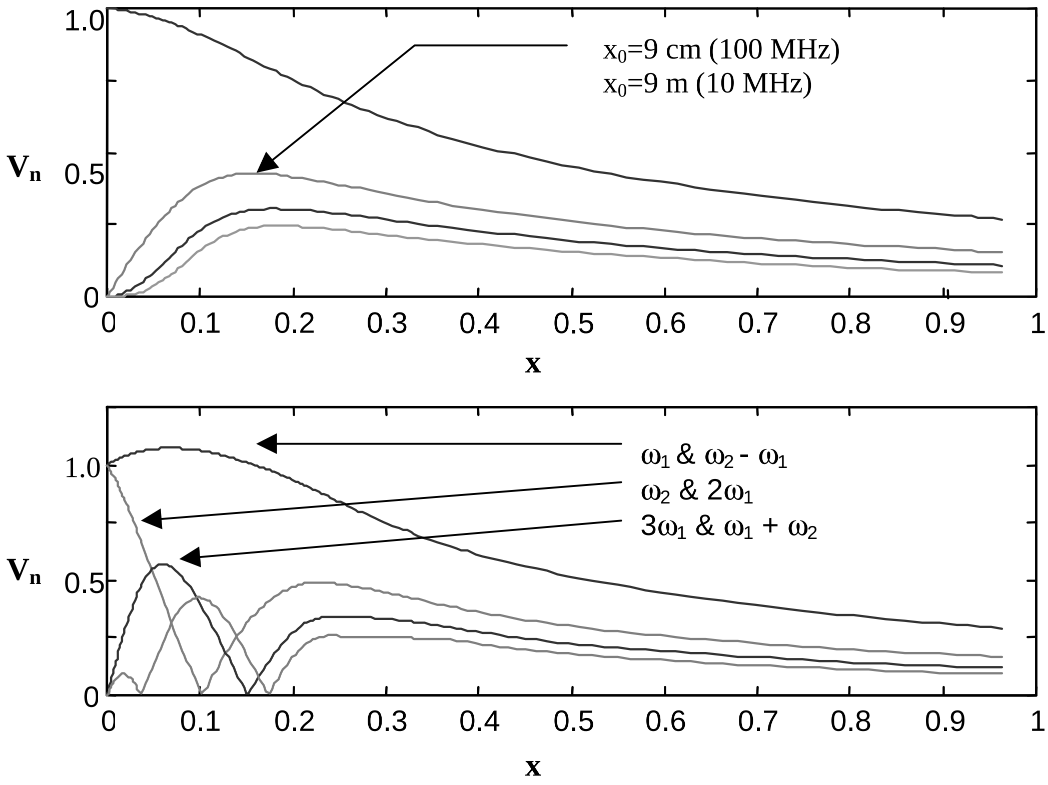
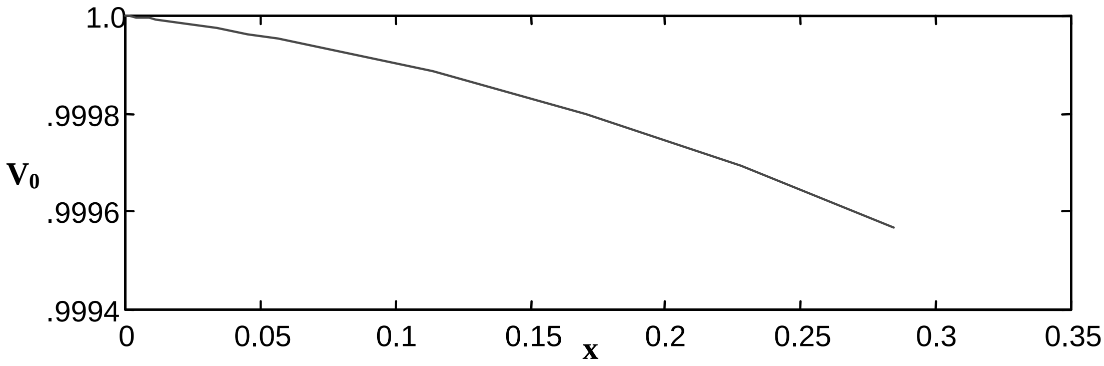


Fig. 1. Top: Harmonic evolution for single frequency source. Bottom: Harmonic evolution for dual frequency source.

The amplitude corresponding to ω_1 initially increases. This is due to the fact that ω_1 is also the difference frequency and as a result, this component receives energy from coupling between ω_1 and $2\omega_1$. The initial rapid decrease in amplitude of the $2\omega_1$ component is a result of this component losing energy to both ω_1, the higher harmonics and attenuation.

When defining a nonlinearity parameter, there are two factors that must be taken into account. First, in order to avoid the difficulties associated with measuring absolute acoustic amplitude, it is convenient too define a parameter that is independent of amplitude. Second, a range of applicability must be defined inside of which the nonlinearity parameter remains independent of amplitude. In order to define a nonlinearity parameter suitable for nonlinear mixing of surface acoustic waves first consider a single frequency source. The fundamental and the first overtone are shown in Figure 2. For small nondimensional propagation distances, the slope of the first overtone is given by,

$$\frac{\Delta V_2}{\Delta x} = F_1 \left| \; S_{11} \right| \; \left(V_1(x=0) \right)^2 \quad , \tag{5}$$



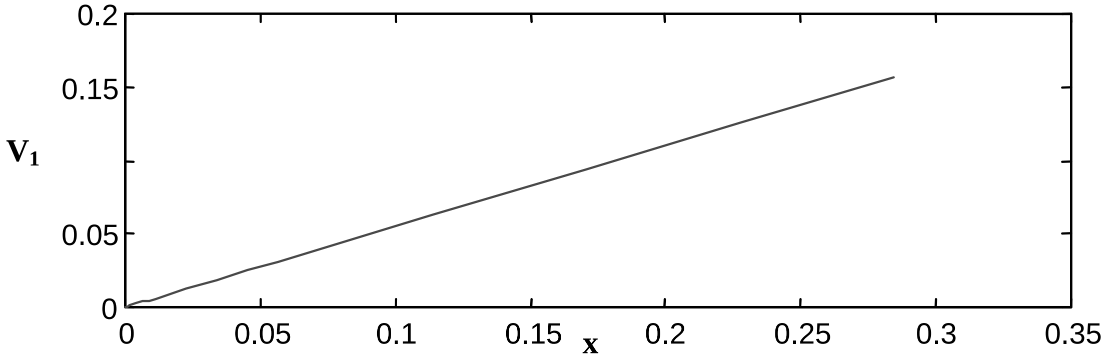
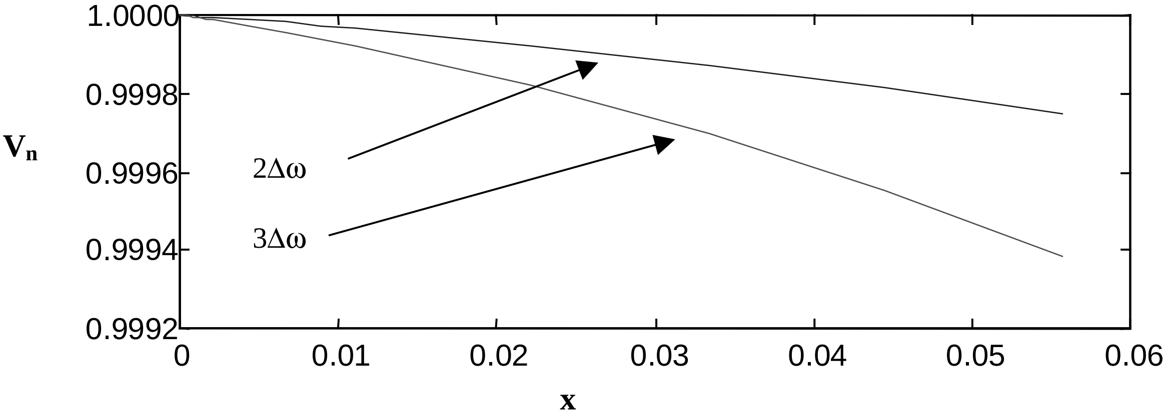


Fig. 2. Top: Decay of the fundamental amplitude for small propagation distances. Bottom: Linear growth of first overtone amplitude for small propagation distances.

to attenuation. Thus, the nonlinearity parameter suggested by this setup is defined as the slope of the first overtone amplitude divided by the square of the fundamental amplitude. It should be noted that this parameter is independent of amplitude for small nondimensional propagation distances, after which attenuation and population of higher harmonics must be taken into account.

For a dual frequency source, (i.e. wave mixing) it is important to judiciously pick the two fundamental frequencies. If for instance, ω_1 and $2\omega_1$ are chosen as shown in the bottom graph in Figure 1, a ratio independent of amplitude can not be defined. This is due to the fact that the amplitude corresponding to ω_1 at small propagation distances is a function of ω_2 since ω_1 is equal to the difference frequency. However, if ω_1 and ω_2 are chosen to be equal to $2\Delta\omega$ and $3\Delta\omega$ respectively, a nonlinearity parameter independent of amplitude can be defined. The two fundamentals, the sum frequency, the difference frequency, and the two second harmonics are shown in Figure 3. As was the case for the single frequency source, the amplitude corresponding to both fundamental frequencies remains roughly constant for small propagation distances. From signal to noise considerations, the slope of the sum frequency is chosen to define a nonlinearity parameter. The slope of the sum frequency is given by

$$\frac{\Delta V_5}{\Delta x} = S_{23}\ F_2\ V_2(x=0)V_3(x=0) \tag{5}$$



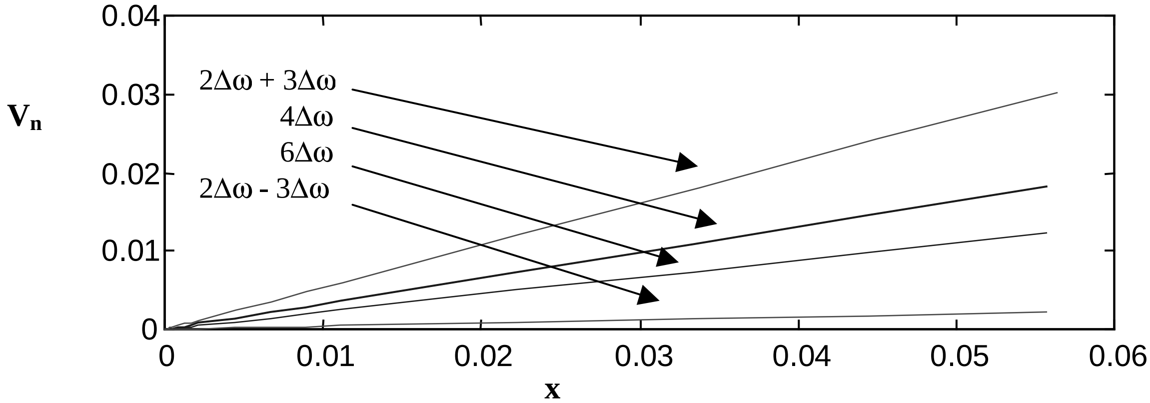


Fig. 3. Top: Initial decay of the amplitude corresponding to both fundamental frequencies. Bottom: Initial linear increase for the amplitudes corresponding to the sum and difference frequencies as well as both second harmonics.

where again F_2 is a function of frequency, density and the second order elastic constants. The nonlinearity parameter suggested by a dual frequency source would be proportional to S_{23}.

PROPOSED EXPERIMENT

We have investigated a number of generation techniques ranging from contact transduction to laser transduction. Due to enhanced reproducibility and easy frequency tuning, our current efforts are focussed on laser generation. A narrow band surface acoustic wave was produced by interfering two generation beams at the surface of the sample. The fringe spacing, which dictates the center frequency of the surface acoustic wave, is given by

$$\Delta = \frac{\lambda}{2\,sin(\varphi/2)} \tag{6}$$

where λ is the optical frequency and ϕ is the angle between the two generation beams. A surface acoustic wave generated in this fashion is shown in figure 4. The out-of-plane displacement was detected using a photorefractive interferometer [9]. A Nd:YAG pulsed laser, pulse duration 10 ns, provided the two generation beams. The fringe spacing is approximately 600 μm, which for the aluminum sample used corresponds to a center frequency of roughly 4.7 MHz.

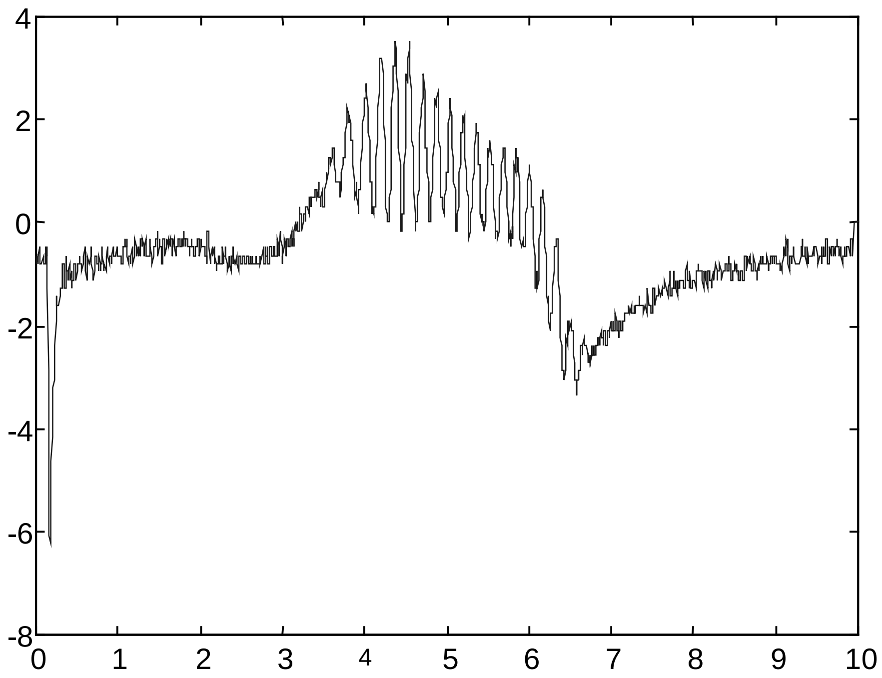


Fig. 4. Out of plane displacement of laser generated narrow band surface acoustic wave.

In order to mimic the conditions imposed by the present theory, an additional narrow-band surface acoustic wave will be generated. The second set of generation beams will be derived from the second harmonic of the Nd.YAG laser. This is done to ensure that any additional optical interference produced at the surface of the sample will be in the form of a traveling wave and will not contribute to the signal at a frequency corresponding to the fringe spacing of the traveling diffraction grating.

CONCLUSIONS

Issues involving sensitivity to background nonlinearity and spatial sensitivity as related to nonlinear acoustics has been discussed. We proposed nonlinear wave mixing of two surface acoustic waves at different frequencies to enhance spatial sensitivity and reduce sensitivity to background nonlinearities. It was shown that if the difference frequency is chosen such that it is not equal to ω_1 or ω_2, a convenient parameter analogous to β can be defined. Initial experimental efforts directed at measuring the nonlinearity parameter associated with wave mixing have been outlined.

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